CHAPTER-1

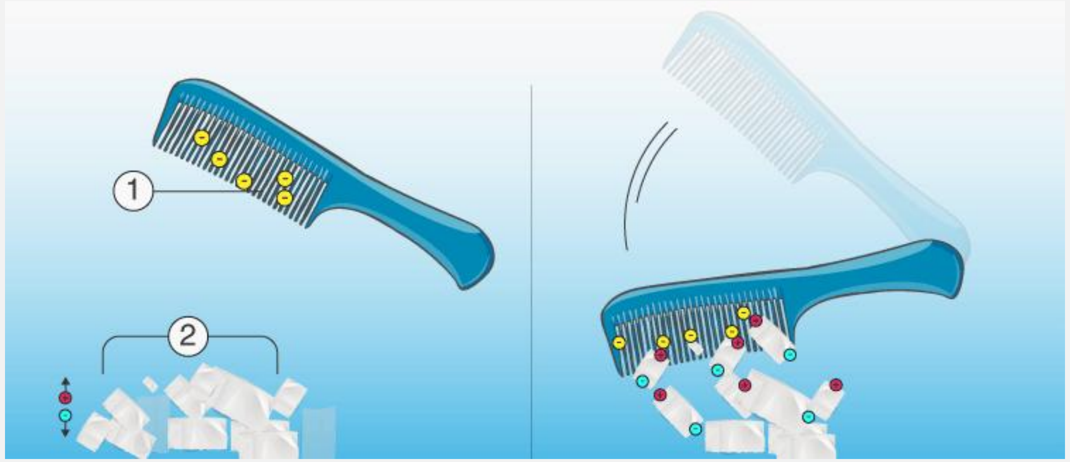
Electric Charges and Fields

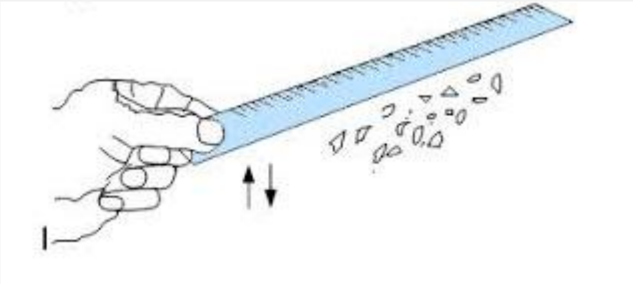
Topics Include

- Electric charges
- Conservation of charge
- Coulomb's law-force between two- point charges
- Forces between multiple charges
- Superposition principle and continuous charge distribution
- Electric field
- Electric field due to a point charge
- Electric field lines
- Electric dipole
- Electric field due to a dipole
- Torque on a dipole in uniform electric field
- Electric flux
- Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

ELECTROSTATICS

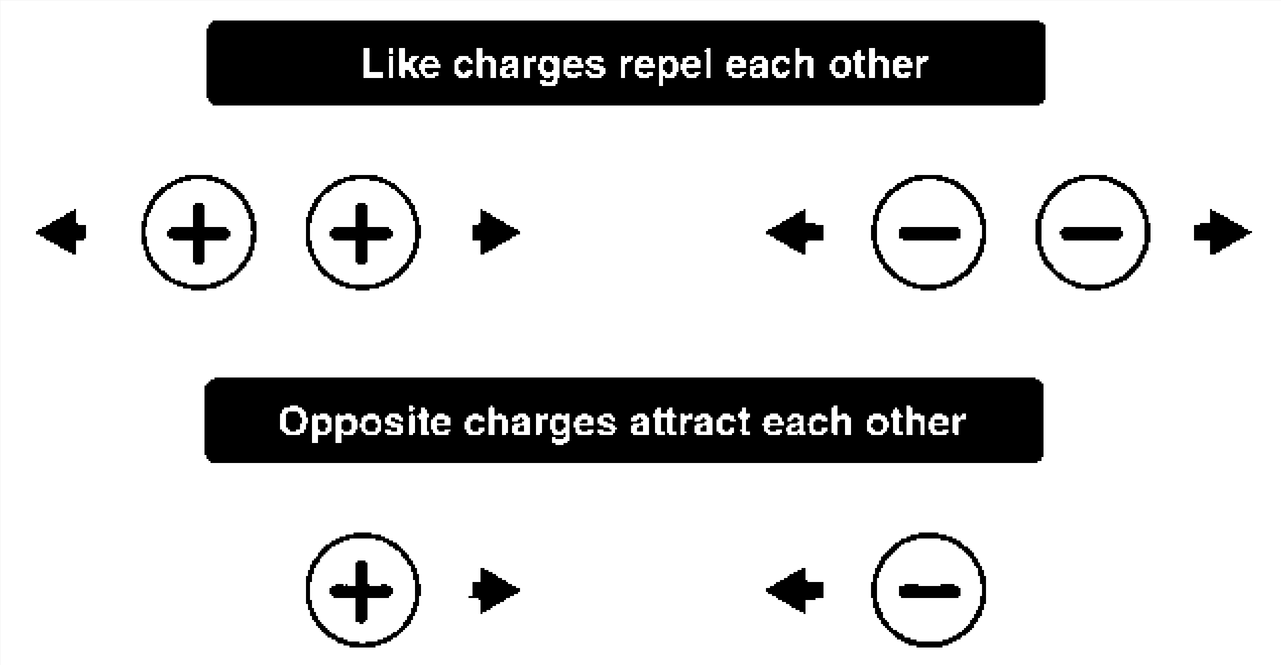
Electrostatics is the branch of Physics which deals with the the study forces, fields, and potentials arising from static charges.





ELECTRIC CHARGES

- Electric charge is a fundamental property of matter that causes it to experience a force in an electric field.



CONSERVATION OF ELECTRIC CHARGE

According to the principle of conservation of charges, the charges are neither created nor destroyed; they are only transferred from one body to the other. For example, when two objects, one has some charge and the other having no charge are made to come in contact with each other, the charge is transferred from the object possessing some charge to the object possessing no charge until the charge is equally distributed over the whole system.

QUANTIZATION OF ELECTRIC CHARGE

According to the principle of quantization of electric charge, all the free charges are integral multiples of a basic predefined unit, which we denote by *e*. Thus, the charge possessed by a system can be given as,

$$\mathbf{q = ne}$$

Where n is an integer (zero, a positive or a negative number) and *e* is the basic unit of charge, that is, the charge carried by an electron or a proton. The value of *e* is 1.6×10^{-19}C.

METHODS OF CHARGING

Charging of a body is the process in which a body gains or loses charged particles (always electrons).

There are three methods:

1. Rubbing (charging by friction)
2. Charging by Conduction
3. Charging by Induction

COULOMB'S LAW

According to **Coulomb's law**, the force of attraction or repulsion between two charged bodies is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. It acts along the line joining the two charges considered to be point charges.

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q_1 q_2}{r^2}$$

where, ε_o is the permittivity of free space, ε_r is also called a dielectric constant of the medium in which the two charges are placed.

absolute permittivity, $\varepsilon = \varepsilon_o \varepsilon_r$

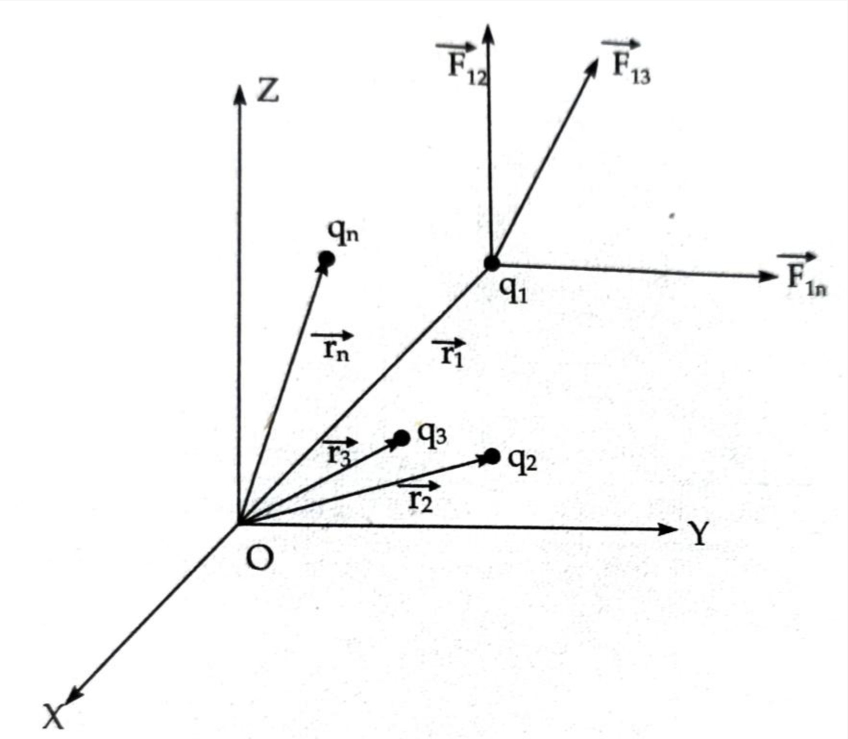
where $\varepsilon_o = 8.854 \times 10^{12}\ C^2 / N\ m^2$

SUPERPOSITION PRINCIPLE AND CONTINUOUS CHARGE DISTRIBUTION

The principle of superposition of electric forces states that when two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges.

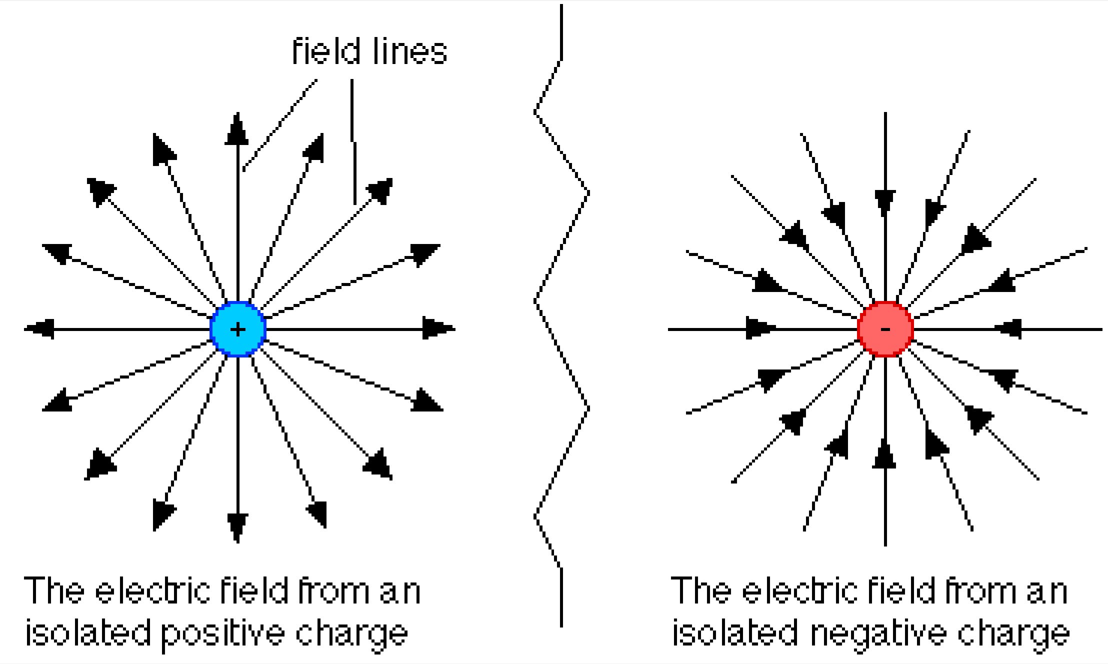
Consider charges q_1, q_2, q_3, ….., q_n located in vacuum, having position vectors r_1, r_2, r_3, …., r_n with respect to origin O. The total force on charge q_1 due to all other charges is

$$\overrightarrow{F_1} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}} + \ldots\ldots + \overrightarrow{F_{1n}}$$



ELECTRIC FIELD

The electric field is defined as the region or space around a charge where an electric force of attraction or repulsion can be experienced.



ELECTRIC FIELD INTENSITY

An electric field intensity is the force per unit charge exerted on a positive test charge at rest at that point. $E = \frac{F}{q}$

Where, E is the electric field, F is the force, q is the charge. $E = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}$

Electric field intensity is a vector quantity. The direction of $\vec{E}$ is that of the force $\vec{F}$ which acts on the positive test charge +q.

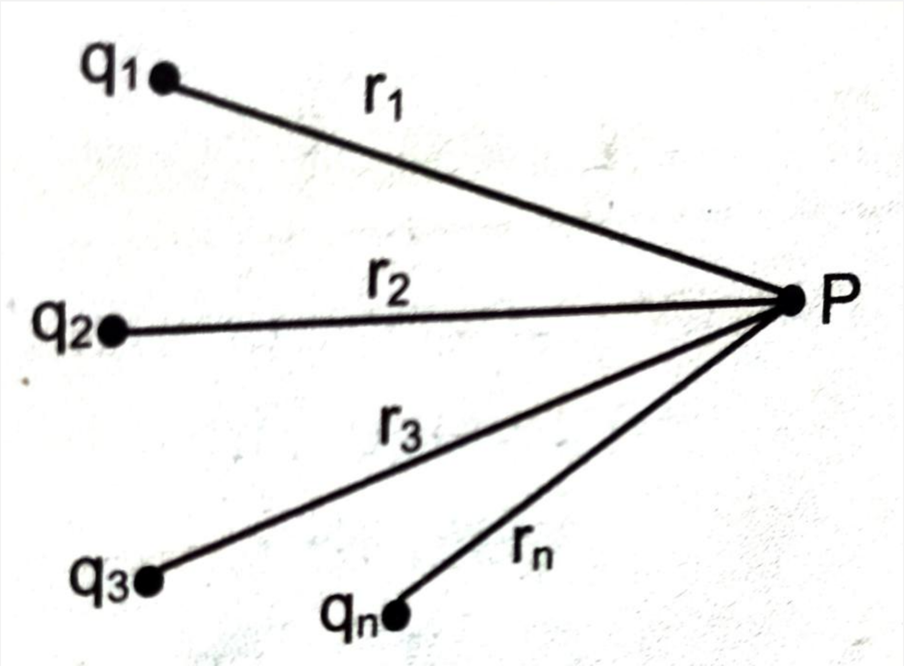
ELECTRIC FIELD DUE TO SEVERAL POINT CHARGE

Let q_1, q_2, q_3, ….., q_n be the n-number of charges in vacuum. Consider a field point P at distances r_1, r_2, r_3, …., r_n from the charges q_1, q_2, q_3, ….., q_n respectively.

Consider E_1, E_2, E_3, ….., E_n be the electric field intensity at a point P due to given charges.

Total electric field intensity at point P is written as the vector sum of electric field due to individual charges, i.e.

$$\vec{\mathbf{E}} = \overrightarrow{\mathbf{E_1}} + \overrightarrow{\mathbf{E_2}} + \ldots\ldots + \overrightarrow{\mathbf{E_n}}$$



ELECTRIC FIELD LINES

Electric Field Lines are a visual representation of the electric field around a charged object or a system of charges. They help illustrate the direction and strength of the electric field in a region of space.

Properties of Electric Field Lines

- The field lines never intersect each other.
- The field lines are perpendicular to the surface of the charge.
- The magnitude of charge and the number of field lines, both are proportional to each other.
- The start point of the field lines is at the positive charge and end at the negative charge.
- For the field lines to either start or end at infinity, a single charge must be used.

ELECTRIC FLUX

Electric flux is the measure of electric lines of force (or electric field lines) passing through a given closed surface. It is a scalar quantity, representing the total number of electric field lines passing through a given surface.

Electric flux is defined as the scalar product of the electric field strength ($\vec{E}$) and area vector ($\overrightarrow{dA}$).

i.e. $d\phi = \vec{E} . \overrightarrow{dA}$

We can get, $\phi = E A \cos\theta$

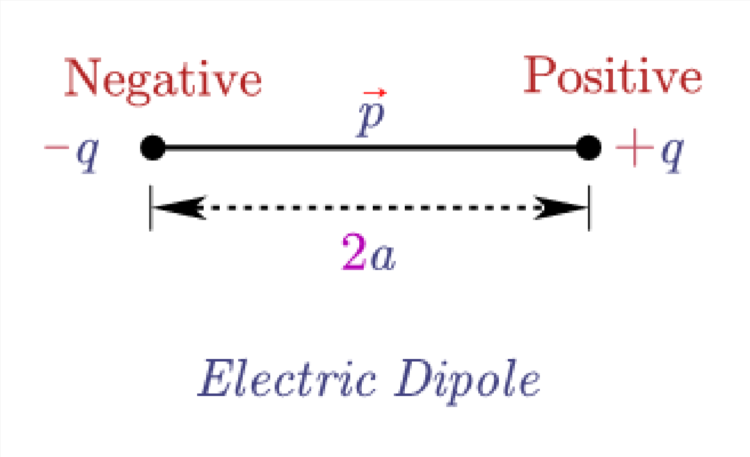
θ = angle between E and the normal vector (A).

- If E is perpendicular to the surface ($\theta=0°$), $\phi = EA$ (maximum).
- If E is parallel to the surface ($\theta=90°$), $\phi = 0$.

ELECTRIC DIPOLE

An electric dipole is a pair of two equal and opposite charges (±q) separated by a small distance (2a). The electric dipole moment is a vector quantity. It has a defined direction which is from the negative charge to the positive charge.

An electric dipole moment is denoted by the symbol "p". We know that electric dipole moment is the product of the magnitude of the charges multiplied by the distance between them.



i.e. $p = q \times (2a)$

ELECTRIC FIELD DUE TO A DIPOLE

The electric field of the pair of charges (–q and q) at any point in space can be found out from Coulomb's law and the superposition principle.

The results are simple for the following two cases:

(i) when the point is on the dipole axis, and

(ii) when it is in the equatorial plane of the dipole, i.e., on a plane perpendicular to the dipole axis through its centre.

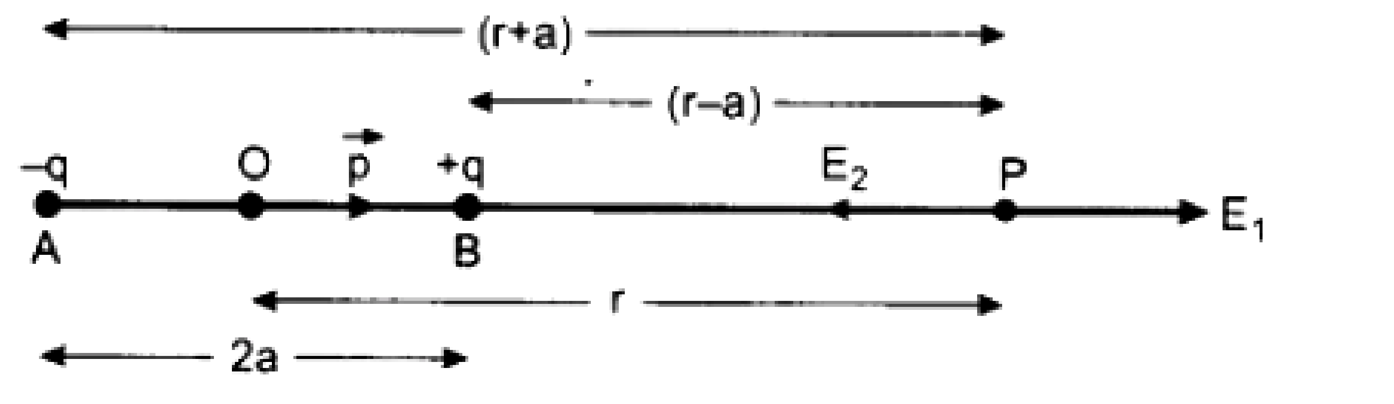
The electric field at any general point P is obtained by adding the electric fields E_{-q} due to the charge –q and E_{+q} due to the charge q, by the parallelogram law of vectors.

ELECTRIC FIELD AT A POINT ON THE AXIAL LINE OF AN ELECTRIC DIPOLE

Let the point P be at distance r from the centre of the dipole on the side of the charge q, then

$E_{-q} = \frac{-q}{4\pi\epsilon_o(r+a)^2}$ and $E_{+q} = \frac{q}{4\pi\epsilon_o(r-a)^2}$

The total field at P is



E = $E_{-q} + E_{+q}$

$$= \frac{q}{4\pi\epsilon_o}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_o}\frac{4\,a\,r}{(r^2-a^2)^2}$$

For r >>a,

$$E = \frac{2q}{4\pi\epsilon_o}\frac{2\,a\,r}{r^4} \qquad E = \frac{1}{4\pi\epsilon_o}\frac{2p}{r^3}$$

ELECTRIC FIELD AT A POINT ON THE EQUATORIAL LINE OF AN ELECTRIC DIPOLE

The magnitudes of the electric fields due to the two charges –q and +q are

$E_{-q} = \frac{q}{4\pi\epsilon_o (r^2+a^2)}$ and $E_{+q} = \frac{q}{4\pi\epsilon_o (r^2+a^2)}$

The total field at P is

$E = (E_{-q} + E_{+q}) \cos\theta$

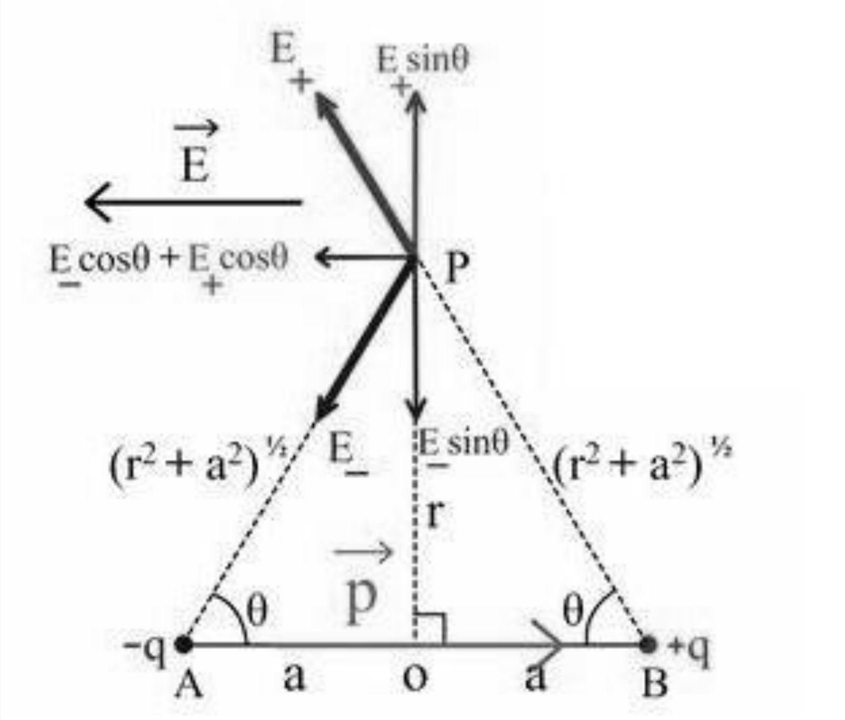
$= 2\frac{q}{4\pi\epsilon_o}\frac{1}{(r^2+a^2)}\cos\theta$

$= 2\frac{q}{4\pi\epsilon_o}\frac{1}{(r^2+a^2)}\frac{a}{(r^2+a^2)^{1/2}}$

$= \frac{2a\,q}{4\pi\epsilon_o}\frac{1}{(r^2+a^2)^{3/2}}$

For r >>a,

$E = \frac{2a\,q}{4\pi\epsilon_o}\frac{1}{r^3} = \frac{1}{4\pi\epsilon_o}\frac{p}{r^3}$



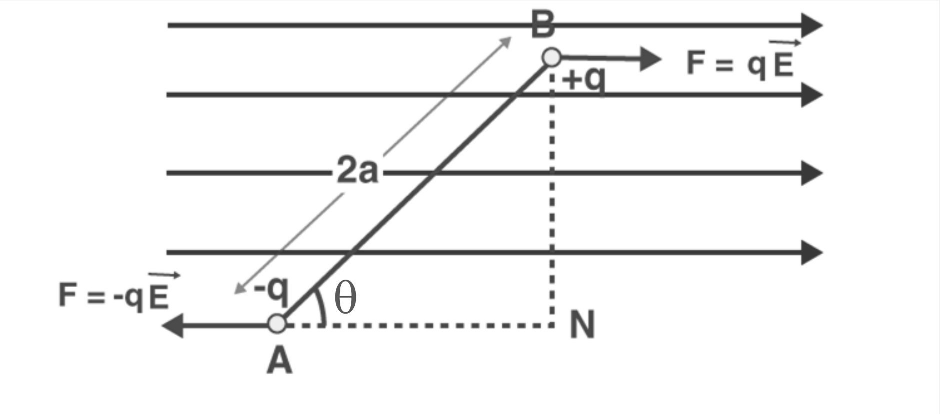
TORQUE ON A DIPOLE IN UNIFORM ELECTRIC FIELD

Consider a dipole of charge q and length 2a placed in a uniform electric field makes an angle $\boldsymbol{\theta}$ with the direction of the electric field.

$\overrightarrow{F_{-q}} = -q\vec{E}$ and $\overrightarrow{F_{+q}} = +q\vec{E}$

The components of force perpendicular to the dipole are:

$-qE \sin\theta$ and $+qE \sin\theta$



The torque on the dipole is given by:

Torque (τ) = Force × distance separating forces

= F × 2a

$\tau = 2a\, qE \sin\theta = pE \sin\theta$

$\therefore \vec{\tau} = \vec{p} \times \vec{E}$

CHARGE DENSITY

- **Linear Charge Density:** The linear charge density is defined as the amount of charge present over a unit length of the conductor. It is denoted by the symbol lambda (λ).

$$\lambda = \frac{q}{L}$$

- **Surface Charge Density:** The surface charge density is defined as the amount of charge present over a unit area of the conductor. It is denoted by the symbol sigma (σ).

$$\sigma = \frac{q}{A}$$

- **Volume Charge Density:** The volume charge density is defined as the amount of charge present over a unit volume of the conductor. It is denoted by the symbol rho (ρ).

$$\rho = \frac{q}{A}$$

GAUSS'S THEOREM

According to Gauss law, the total flux linked with a closed surface is $1/\varepsilon_0$ times the charge enclosed by the closed surface.

i.e. $\phi = \frac{1}{\epsilon_0} q_{net}$

From the definition of flux,

$\phi = \oint \vec{E}.\overrightarrow{dA}$

Then, $\oint \vec{E}.\overrightarrow{dA} = \frac{1}{\epsilon_0} q_{net}$