# CHAPTER-2

# Electrostatic Potential and Capacitance

# Topics Include

- Electric potential
- Potential difference
- Electric potential due to a point charge
- a dipole and system of charges
- Equipotential surfaces
- Electrical potential energy of a system of two-point charges and of electric dipole in an electrostatic field
- Conductors and insulators
- Free charges and bound charges inside a conductor
- Dielectrics and electric polarization
- Capacitors and capacitance
- Combination of capacitors in series and in parallel
- Capacitance of a parallel plate capacitor with and without dielectric medium between the plates
- Energy stored in a capacitor (no derivation, formulae only)

# ELECTRIC POTENTIAL

**Electric potential** is defined as the amount of work/energy needed per unit of electric charge to move the charge from a reference point to a specific point in an electric field. Electric potential is a scalar quantity.

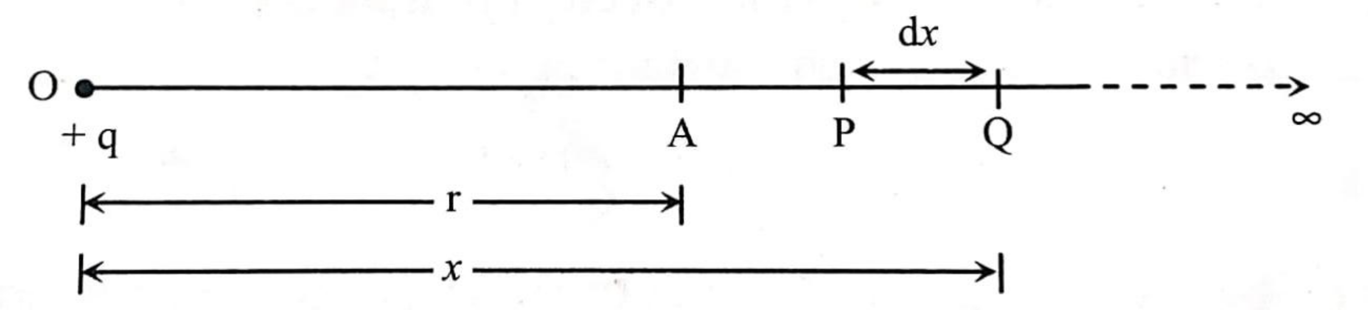
Electric potential = $\frac{\text{Work done}}{\text{Charge}}$

i.e. $V = \frac{W}{q}$

It's unit is Volt (V) or J/C.

# ELECTRIC POTENTIAL DUE TO A POINT CHARGE

Electric Potential at a point in the electric field is defined as the work done in moving a unit positive charge from infinity to that point against the electrostatic force.




$dW = Fdx\cos\theta = F.dx.\cos 180°$

$F = k\frac{Qq}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{x^2}$

Then, $dW = \frac{1}{4\pi\epsilon_0}\frac{Qq}{x^2}.dx.(-1)$

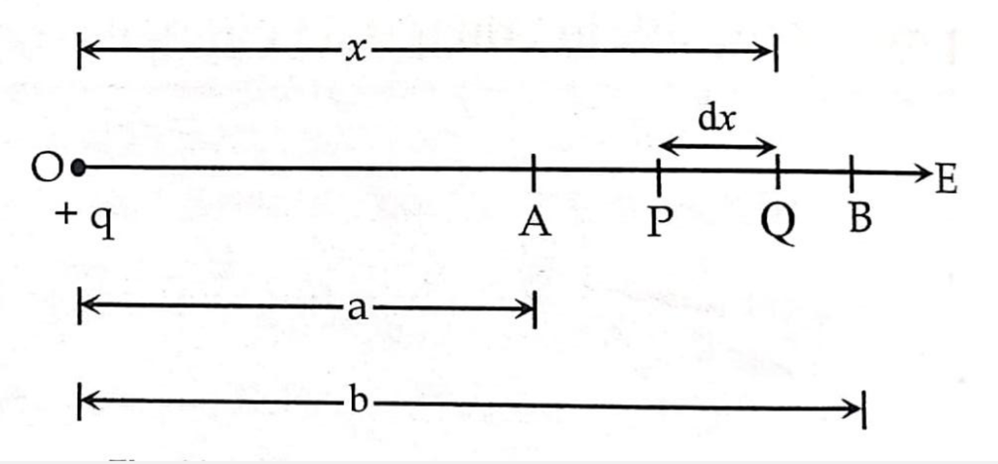
Therefore, $W = \int_{\infty}^{r}\frac{-1}{4\pi\epsilon_0}\frac{Qq}{x^2}dx = \frac{-Qq}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{1}{x^2}dx = \frac{-Qq}{4\pi\epsilon_0}\left[\frac{x^{-2+1}}{-2+1}\right]_{\infty}^{r} = \frac{Qq}{4\pi\epsilon_0}\left[\frac{1}{r}-\frac{1}{\infty}\right]$

$W = \frac{Qq}{4\pi\epsilon_0}\frac{1}{r}$

Hence, $V = = \frac{W}{q} = \frac{Q}{4\pi\epsilon_0 r}$

# POTENTIAL DIFFERENCE

Potential difference between two points is defined as the amount of work done in bringing a unit positive charge from one point to another. The potential difference between two points A and B in the electric field is denoted by $V_{AB}$ or simply V.




As we have, $dW = \frac{1}{4\pi\epsilon_0}\frac{Qq}{x^2}.dx.(-1)$

$$W = \int_b^a \frac{-1}{4\pi\epsilon_0}\frac{Qq}{x^2}dx \quad = \frac{Qq}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Hence, $V = = \frac{W}{q} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$

# ELECTRICAL POTENTIAL ENERGY

The electric potential energy of any given charge or system of changes is defined as the total work done by an external agent in bringing the charge or the system of charges from infinity to the present configuration.

Since electrostatic force is conservative, this work gets stored in the form of potential energy of the system. The potential difference between two points is independent of the path through which it is displaced. But depends only on the initial and final positions. Thus, the potential energy of a system of two charges $q_1$ and $q_2$ is determined by integrating the elementary workdone in the varying field regions.

Electric Potential Energy (U) = $\int_{\infty}^{r} \frac{-1}{4\pi\epsilon_0} \frac{q_1 q_2}{x^2} dx \quad = \frac{-q_1 q_2}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{1}{x^2} dx \quad = \frac{-q_1 q_2}{4\pi\epsilon_0} \left[ -\frac{1}{r} \right]$

$$\therefore U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

# ELECTRON VOLT

In atomic scale, joule is a very large unit for dealing with energies of electrons, atoms etc. To deal with such scale, smaller unit of energy known as electron volt is used. Electron volt denoted by eV is simply, 1 volt multiplied by the magnitude of charge of one electron.

i.e. $1\ eV = (1.6 \times 10^{-19}\ C)\ (1\ volt)$

$= 1.6 \times 10^{-19}\ J$

Thus, one electron volt is the potential energy gained or lost by an electron in moving through a potential difference of 1 volt.

# POTENTIAL DUE TO AN ELECTRIC DIPOLE

$V_{+q} = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1}$ and $V_{-q} = \frac{1}{4\pi\epsilon_0}\frac{-q}{r_2}$

$V = V_{+q} + V_{-q} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

Let, AN = $a\sin\theta$ and ON = $a\cos\theta$

$r_1^2 = (AN)^2 + (PN)^2 = (a\sin\theta)^2 + (OP-ON)^2$

$= (a\sin\theta)^2 + (r - a\cos\theta)^2$

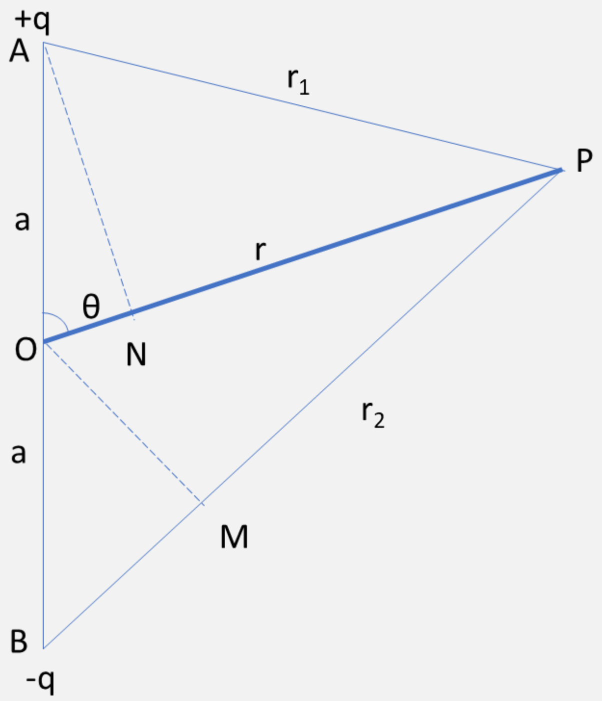
$r_1^2 = a^2 + r^2 - 2ar\cos\theta$

For r >> a, $r_1^2 = r^2\left(1 - \frac{2a\cos\theta}{r}\right)$

$r_1 = r\left(1 - \frac{2a\cos\theta}{r}\right)^{1/2}$

$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a\cos\theta}{r}\right)^{-1/2}$

$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a\cos\theta}{r}\right)$




**Using the Binomial theorem**

**If $\left(1+\frac{a}{b}\right)^n$ and a << b, then**

$\left(1+\frac{a}{b}\right)^n = \left(1+\frac{na}{b}\right)$

# POTENTIAL DUE TO AN ELECTRIC DIPOLE

Similarly, $r_2^2 = a^2 + r^2 + 2ar\cos\theta$

For $r >> a$, $\quad r_2^2 = r^2\left(1 + \frac{2a\cos\theta}{r}\right)$

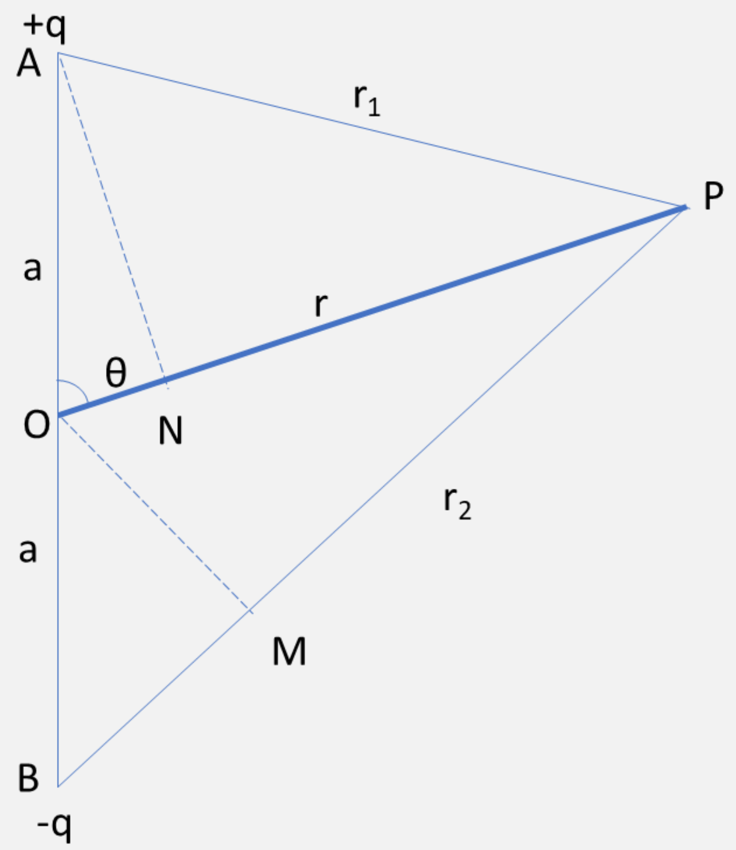
$$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a\cos\theta}{r}\right)$$

Substituting in $V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$,

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\left(1 + \frac{a\cos\theta}{r}\right) - \frac{1}{r}\left(1 - \frac{a\cos\theta}{r}\right)\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{2a\cos\theta}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$$



# POTENTIAL DUE TO AN ELECTRIC DIPOLE

We have, $V = \frac{1}{4\pi\epsilon_0} \frac{p\cos\theta}{r^2}$

Case 1: A point on the axial line [θ = 0°]

$$\therefore V = \frac{1}{4\pi\epsilon_0} \frac{p\cos 0}{r^2}$$

$$= \frac{p}{4\pi\epsilon_0 r^2}$$

Case 2: A point on the equatorial line [θ = 90°]

$$\therefore V = \frac{1}{4\pi\epsilon_0} \frac{p\cos 90}{r^2}$$

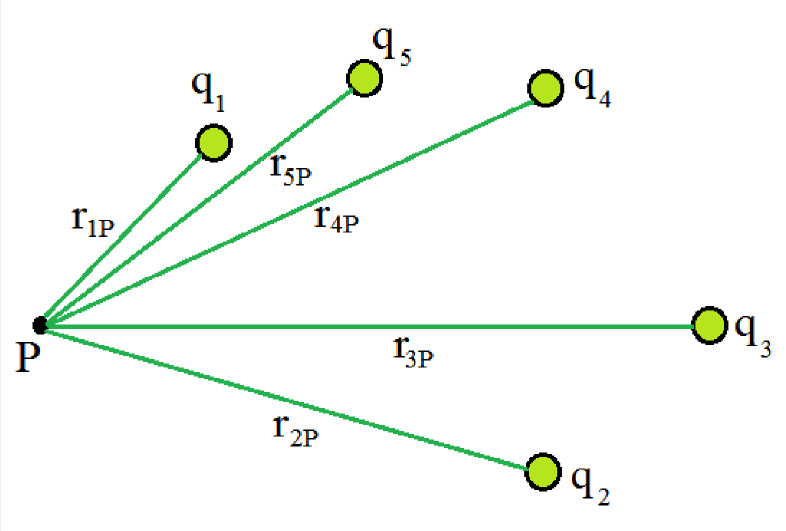
$$= 0$$

# POTENTIAL DUE TO SYSTEM OF CHARGES

Consider a system of charges $q_1$, $q_2$ ,…, $q_n$ at a distance of $r_{1P}$, $r_{2P}$ ,…, $r_{nP}$ from a point respectively.

The potential at P due to individual charges will be given by

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{1P}},\ V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{2P}}\ \ldots\ldots..,\ V_N = \frac{1}{4\pi\epsilon_0}\frac{q_n}{r_{nP}}$$




The net potential due to these point charges is given by,

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{1P}} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{2P}} + \cdots + \frac{1}{4\pi\epsilon_0}\frac{q_n}{r_{nP}}$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \cdots + \frac{q_n}{r_{nP}}\right]$$

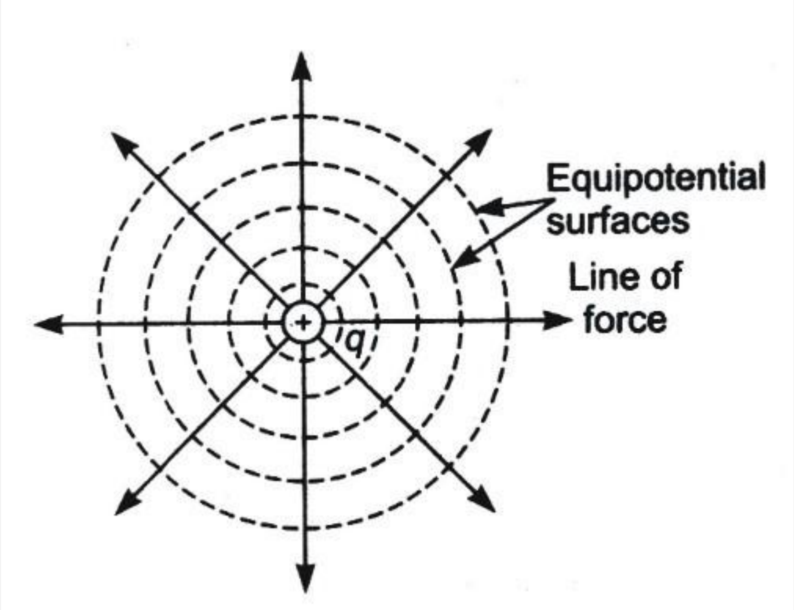
# EQUIPOTENTIAL SURFACES

In short, the surface in an electric field in which the electric potential is same in every point of it is known as equipotential surface.

If a point charge is moved from point $V_A$ to $V_B$ in an equipotential surface, then the work done in moving the charge is given by

$W = q_0(V_A - V_B)$

As $V_A - V_B$ is equal to zero, the total work done is $W = 0$.



# POTENTIAL GRADIENT AND ELECTRIC FIELD

The change in electric potential with respect to distance in known as potential gradient.

By definition of work done,

$\Delta V = \vec{E} . \Delta\vec{r}$ or, $\Delta V = E\Delta r \cos\theta$

Since the unit positive charge is moved towards +q against electrostatic force, $\theta = 180°$, $\Delta V = -E\Delta r$

or, $E = -\frac{\Delta V}{\Delta r}$

Therefore, the magnitude of electric field intensity is numerically equal to the potential gradient. For two surfaces with a p.d. V between them and at a distance d apart, electric field

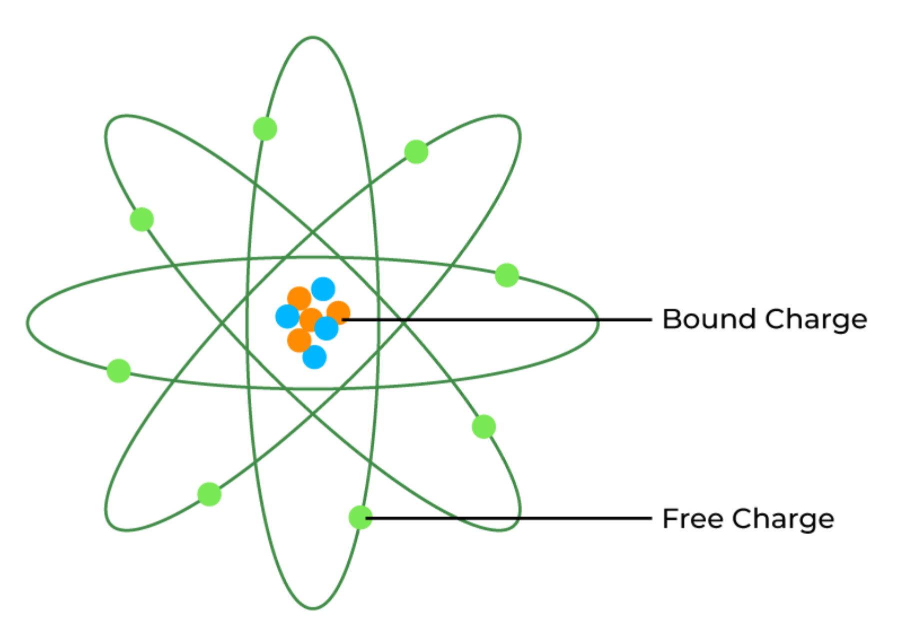
$$E = \frac{V}{d}$$

# ELECTROSTATICS OF CONDUCTORS

1. Inside a conductor, electrostatic field is zero
2. At the surface of a charged conductor, electrostatic field must be normal to the surface at every point
3. The interior of a conductor can have no excess charge in the static situation
4. Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface
5. Electric field at the surface of a charged conductor is $\frac{\sigma}{\epsilon_0}$
6. Electrostatic shielding i.e. electric field inside the cavity is always zero.

# FREE CHARGES AND BOUND CHARGES INSIDE A CONDUCTOR

Free charges are the charges that are free to move within the conductor (typically electrons in a metal).



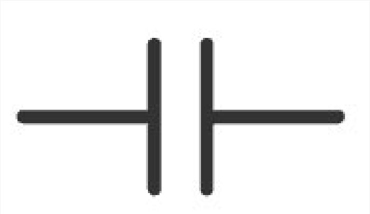

Bound charges are the charges that are not free to move and are associated with the atoms or molecules of the material (e.g., electrons bound to nuclei in insulators). However, in a perfect conductor, the concept of bound charges is less relevant because all charges are free to move.

# CAPACITORS AND CAPACITANCE

A capacitor is a two-terminal electrical device that can store energy in the form of an electric charge.

Any conductor which can hold charge is a capacitor. More precisely, a capacitor is a device designed to store electric charge and hence the electric energy in the electric field.

It is found that potential difference (V) is directly proportional to the potential difference (V) applied across it, then



$Q \propto V$

or, $Q = CV$ $\Rightarrow C = \frac{Q}{V}$

where C is a proportionality constant known as the capacitance of the capacitor.

Capacitance of a capacitor is defined as the charge stored per unit potential difference. It is the measure of how much charge a capacitor can hold per applied voltage. It's unit is Farad (F) which is $CV^{-1}$.

# PARALLEL PLATE CAPACITOR

In parallel plate capacitor, the electric field between the plates is

$E = \frac{\sigma}{\epsilon_0}$ where σ is the surface charge density of capacitor plates.

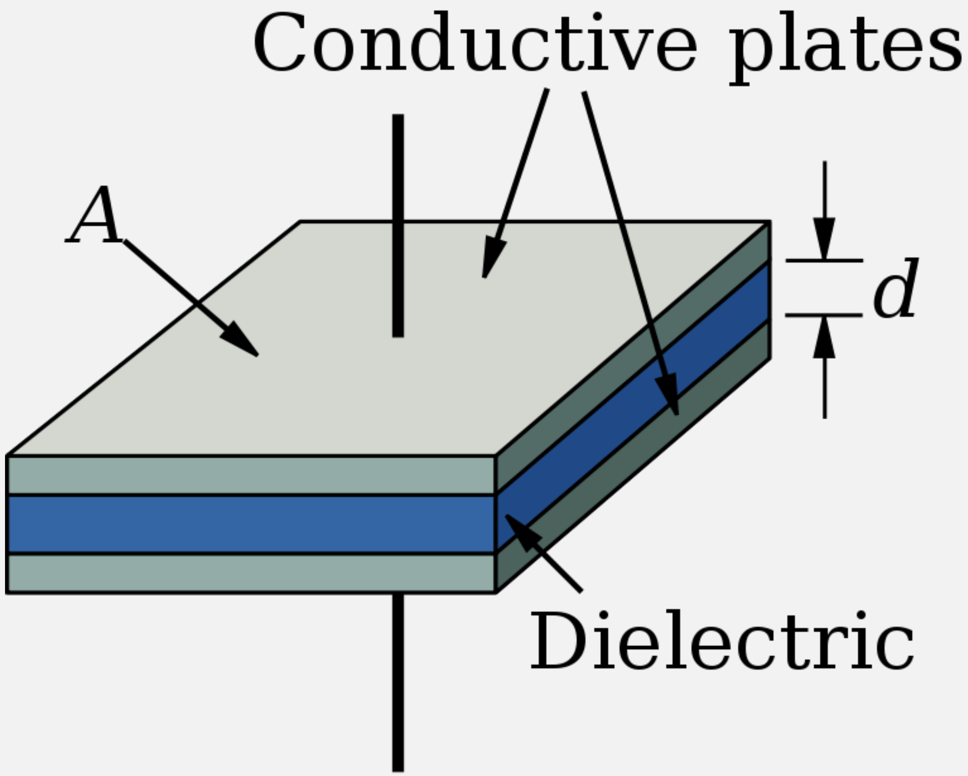
Also, $E = \frac{V}{d}$

Then, $\frac{V}{d} = \frac{\sigma}{\epsilon_0}$ $\implies V = \frac{\sigma d}{\epsilon_0}$

The total charge on a plate is $Q = \sigma A$

Hence, $C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma d}{\epsilon_0}}$

$\therefore C = \frac{\epsilon_0 A}{d}$



# EFFECT OF DIELECTRIC ON CAPACITANCE

When the two plates have a vacuum between them, potential difference across the capacitor can be given as, $V = Ed = \frac{\sigma}{\epsilon_0} d$

The capacitance $C_0$ of the capacitor can thus be given as, $C_0 = \frac{\epsilon_0 A}{d}$

Let us insert a dielectric between the plates such that it fully occupies the space between the plates. As the dielectric enters the field between the plates, it gets polarized by the field, and the charges get arranged such that they act as two charged sheets with a surface charge density of $\sigma_p$ and $-\sigma_p$.

The net surface charge density then becomes equivalent to $\pm(\sigma - \sigma_p)$

The potential difference across the capacitor can thus be given as,

$$V = Ed = \frac{\sigma - \sigma_p}{\epsilon_0} d$$

# EFFECT OF DIELECTRIC ON CAPACITANCE

CONTD.

In the case of linear dielectrics, we can say that $\sigma_p$ is proportional to $E_0$ and hence it is proportional to σ. Thus, we can say that the value ($\sigma - \sigma_p$) is also proportional to σ. Mathematically, $\sigma - \sigma_p = \frac{\sigma}{K}$

Where K is a constant, whose value depends upon the dielectric medium selected. The potential energy across the capacitor can this be written as,

$$V = \frac{\sigma d}{\epsilon_0 K} = \frac{Qd}{A\epsilon_0 K}$$

And the capacitance $C_m$ between the plates can be given as,

$$C_m = \frac{\epsilon_0 KA}{d}$$

# DIELECTRIC CONSTANT

CONTD.

$\varepsilon_0$K is the permittivity of the medium, which can also be given as,

$$\epsilon = \epsilon_0 \mathrm{K}$$

Here the value K is the permittivity of the medium such that, for a given medium,

$$\mathrm{K} = \frac{\epsilon}{\epsilon_0}$$

And the ratio of the capacitance of the capacitor with a dielectric medium to the capacitor with a vacuum between the plates can be given as,

$$\mathrm{K} = \frac{\mathrm{C_m}}{\mathrm{C_0}}$$

Hence, Dielectric constant of a substance is defined as the ratio of the capacitance of a capacitor with dielectric to its capacitance without dielectric. It is denoted by K or $\epsilon_\mathrm{r}$. It is also called relative permittivity of the substance.

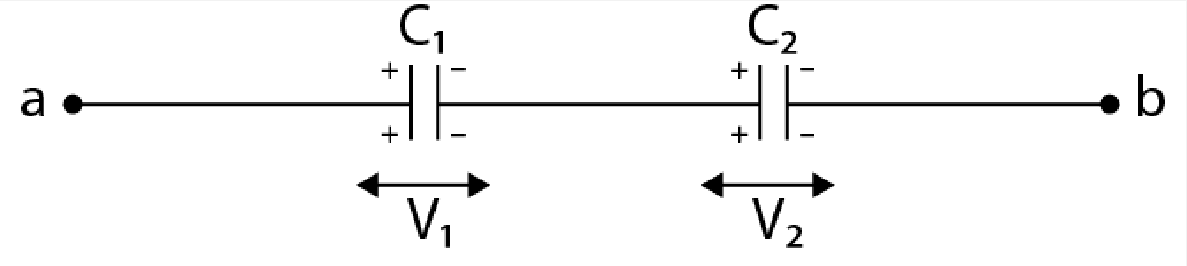
# COMBINATION OF CAPACITORS IN SERIES

When capacitors are connected in series, the magnitude of charge Q on each capacitor is the same. The potential difference across $C_1$ and $C_2$ is different, i.e., $V_1$ and $V_2$.

The total potential difference across combination is:

$$V = V_1 + V_2 \qquad \Longrightarrow V = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$




The ratio Q/V is called the equivalent capacitance C between points a and b. The equivalent capacitance C is given by: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

In the case of more than two capacitors, the relation is:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \ldots\ldots\ldots\ldots$$

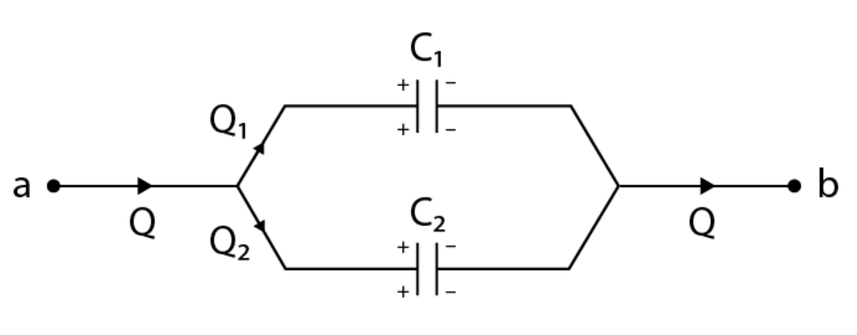
# COMBINATION OF CAPACITORS IN PARALLEL

When capacitors are connected in parallel, the potential difference V across each is the same and the charge on $C_1$ and $C_2$ is different, i.e., $Q_1$ and $Q_2$.

The total potential difference across combination is:

$Q = Q_1 + Q_2$ $\Longrightarrow Q = C_1V + C_2V$

$$\frac{Q}{V} = C_1 + C_2$$




The ratio Q/V is called the equivalent capacitance C between points a and b.

The equivalent capacitance C is given by: $C = C_1 + C_2$

In the case of more than two capacitors, the relation is:

$$C = C_1 + C_2 + C_3 + C_4 + \ldots\ldots\ldots\ldots$$

# ENERGY STORED IN A CAPACITOR

Let the capacitance of a conductor be C and charge on it be initially zero. After connection to battery, let it acquires charge +Q at potential difference V. Then, $C = \frac{Q}{V}$

If the battery delivers a small amount of charge dQ at a constant potential V, then the work done is dW = V dQ

Now, the total work done in delivering a charge of an amount q to the capacitor is given by

$$W = \int_0^Q V\,dQ = \int_0^Q \frac{Q}{C}\,dQ = \frac{1}{C}\int_0^Q Q\,dQ = \frac{1}{C}\left[\frac{Q^2}{2} - 0\right]$$

$$\therefore W = \frac{1}{2}\frac{Q^2}{C}$$

This work is stored in the capacitor as the electric potential energy U.

Also, we can write $U = W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$

# ENERGY DENSITY

It is often useful to consider the stored energy to be localized in the electric field between the capacitor plates. So, the expression of energy density is expressed in terms of electric field intensity. Energy density of a capacitor is defined as the total amount of energy stored per unit volume.

$\therefore$ Energy Density ($u$) = $\frac{U}{\text{Volume}} = \frac{1}{2}\frac{\text{CV}^2}{\text{A.d}}$

where, A = surface area of a capacitor plate

d = separation between two plates

Also, C = $\frac{\epsilon \text{A}}{\text{d}}$ and V = Ed

$$\therefore u = \frac{1}{2}\frac{\frac{\epsilon \text{A}}{\text{d}}(\text{Ed})^2}{\text{A.d}} = \frac{1}{2}\epsilon \text{E}^2$$